Cucker–Smale's flocking model with a singular communication weight

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Abstract

The Cucker–Smale's flocking model with a smooth communication weight was thoroughly studied and is well known. However with a singular communication weight it proves to be more difficult and interesting.

The aim is to present the Cucker-Smale's model describing collective self-driven motion of self-propelled particles, such as flocking of birds, shooling of fishes or concentration of network activity. In the discrete case, given N particles, the motion of each follows according to a system of equations

$$\dot{x}_i(t) = m_i(t),$$

$$\dot{m}_i(t) = \frac{1}{N}\sum_{j=1}^{N} \psi_{cs}(|x_j - x_i|)(m_j - m_i),$$

where $x_i(t), m_i(t)$ are respectively the position and velocity of i-th particle at the time t, while ψ_{cs} is the *communication weight* (or *communication rate*).
The most commonly considered communication weight is

$$\psi_{cs}(s) = \frac{K}{(1+s^2)^{\beta}},\ \beta \geq 0,\ K > 0.$$

Our goal is to present the difficulties, that arise from replacing ψ_{cs} with a singular weight

$$\psi_{sing}(s) = |s|^{-\alpha},\ 0 < \alpha < d$$

and the first results involving existence and uniqueness. If time permits we will also present the continuous Cucker–Smale's flocking model.

Keywords: Cucker–Smale's flocking model, singular communication weight.

References

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